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BIVARIATE OPTIMAL REPLACEMENT POLICY FOR A DETERIORATING REPAIRABLE SYSTEM USING ARITHMETICO- GEOMETRIC PROCESS (AGP)

KEY WORDS: Renewal reward theorem, Replacement policy N^* , Long-run average cost per unit time, bivariate replacement policy, geometric process, arithmetic-geometric process.

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ABSTRACT

This paper studies an optimal bivariate replacement policy for a deteriorating repairable system by assuming that the successive operating times of the system form - a decreasing Arithmetic-Geometric Process (AGP) while the successive repair times of the system form - an increasing AGP. Under these assumptions we study an optimal replacement policy (T, N) for a cold standby repairable system under which we replace the system when the operating time of component 1 reaches T and the number of repairs of component 1 reaches N whichever occurs first. Under some conditions, an explicit expression C(N) for the long run average cost per unit time of the system is derived and determined corresponding optimal replacement N^* such that the long run average cost is minimum. Finally numerical results are provided to highlight the theoretical results.

2.1 INTRODUCTION

In maintenance problems, it is assumed that a failure system after repair will function "as good as new" and the repair times are neglected so that the successive working times generate a renewal process. These models are called perfect repair models. However, it is not always true for a deteriorating system, because the system after repair can't be 'as good as new' condition. Under this assumption, Barlow and Hunter [1] introduced a minimal repair model, in which a failed system after repair will function again, but with the same failure rate and with the same effective age as it was prior to failure. Thereafter, an imperfect repair model was first introduced by Brown and Proschan [2,3] under which a repair with probability p is a perfect repair and with probability $q = 1 - p$ is a minimal repair. Much research work has been carried out in this direction by Park[12], Block et al[4,5] and others. However, in practice, due to the ageing effect and accumulated wearing many systems deteriorate. For a deteriorating system, it is reasonable to assume that the successive operating times after repair are stochastically decreasing while the consecutive repair times after failure are stochastically increasing. Thus a deteriorating system can't work any longer neither can it be repaired nor retained any more.

To model such simple repairable deteriorating system, Lam [9,11] first introduced a geometric process repair model under the assumption that the system after repair is not 'as good as new' and the successive working times $\{x_n, n=1,2, \dots\}$ of a system form a non - increasing geometric process and the consecutive repair time $\{y_n, n=1,2, \dots\}$ form a non - decreasing geometric process. Under these assumptions, he developed two kinds of replacement policy- one based on the working age T of the system and the other based on the number of failures N of the system. He derived explicit expressions for the long - run average cost per unit time under both policy and also proved that the optimal replacement policy N^* is better than the optimal policy T^* . Later Finkelstein [6] presented a general repair model based on scale transformation to generalize Lam's work. Lam [10] studied an optimal replacement model for deteriorating system and determine the optimal replacement policy N^* such that the expected discounted reward cost is minimum. Zhang et al [18] presented an optimal replacement policy for a deteriorating production system with preventive maintenance which is parallel to the geometric process replacement policy introduced by Lam [10]. Later Zhang [16] generalized Lam's work by a bivariate replacement policy (T, N) under which the system is replaced when working age of the system reaches T and the failure number of the system reaches N whichever occurs first. Other replacement policies under the geometric process repair model were reported by Stage and Zuckerman [15] and Zhang [17, 19, 20], Leung [7,8].

The above research work pointed out to a one component repairable system. However in practical applications for improving

the reliability and raising the availability of the system, the stand by techniques are usually used. Thus Zhang [17] applied the geometric process repair model to a two - identical component cold standby repairable system with one repairman and assumed that each component after repair is 'not as good as new'. Using the geometric process, he studied an optimal replacement policy N^* such that the long run average reward cost is minimum.

Later to generalize Zhang's [16,17] work Zhang and Wang [21] introduced a bivariate optimal repair replacement model using geometric process for a cold standby repairable system with one repairman. It is assumed that the system after repair is not 'as good as new'. Under this assumption they studied an optimal replacement policy (T, N) under which the system is replaced when working age of the system reaches T and the number of failures of the system reaches N whichever occurs first. They derived an explicit expression for the long run average cost per unit time of the system and corresponding optimal replacement policy (T, N)* such that the long - run average cost per unit time of the system is minimum analytically (or) numerically.

The purpose of this study is to generalize the Zhang and Wang [21] work by applying the arithmetic-geometric process repair model to a two component cold standby system with one repairman. An arithmetic-geometric process approach is considered to be more relevant, realistic and direct to the modeling of the deteriorating system in maintenance problems that are encountered in most situations other than perfect or minimal repair models. In practice, the data of successive inter event times usually exhibit a trend. They may be modeled using a non - homogeneous Poisson process (NHPP) in which the rate of occurrence of failure at time 't' is a function of 't'. Thus NHPP is a popular approach to minimal repair model where repair time is assumed to be negligible. The NHPP in which the rate of occurrence occurs time is monotone can be provided at least good first order model for deteriorating system. Based on this understanding an AGP (Average geometric process) approach here is more relevant, realistic and direct to the modeling of the maintenance problems of deteriorating systems, because AGP is a special case of NHPP. Therefore for a deteriorating system, it is reasonable to assume that the successive operating times of the system form - a decreasing AGP while the successive repair times of the system form - an increasing AGP. Under these assumptions we study an optimal replacement policy (T, N) for a cold standby repairable system under which we replace the system when the operating time of component 1 reaches T and the number of repairs of component 1 reaches N whichever occurs first. We derive explicit expression C(N) for the long run average cost per unit time of the system and determine corresponding optimal replacement N^* such that the long run average cost is minimum. Finally numerical results were provided to highlight the theoretical results.

Before modeling we make use the definition of AGP As follows (see [7]):

Arithmetico – Geometric Process(AGP)

A Stochastic process $\{H_n, n=1,2,\dots\}$ is an arithmetico-geometric process if there exists some real number d and some real positive number r , such that $\{[H_n + (n-1)d]r^{n-1}, n=1,2,\dots\}$ form a renewal process (RP), where the two parameters d and r called the common difference and common ratio of the AGP respectively.

Clearly if $r>1$ and $d[0,/(n-1)r^{n-1}]$, where $n=2,3,\dots$ and μ_1 is the mean of the first random variable H_1 , then AGP is called a decreasing AGP. If $d<0$ and $0<r<1$, then the AGP is called increasing AGP. If $d=0$ and $r=1$, then the AGP reduces to a renewal process.

2 MODEL

In this section, we develop a model for a bivariate optimal replacement policy (T, N) for a cold standby system using AGP exposing to Weibull failure law in such way that the long run average cost per unit time is minimum. The following are the assumptions.

Assumptions

1. At the beginning, both the components are new and component 1 is in the working state while the other is in cold standby state.
2. The two components appear alternatively in the system i.e., when working component fails immediately the standby component begins to work and the failed one is repaired by the repairman. Whenever the repair of the failed one is completed, it becomes cold standby. If one fails and the other is still under repair then the system breaks down.
3. A component in the system is replaced some time by an identical one and the replacement time is negligible.
4. The components after repair are not as good as new. The time interval between the completion of the $(n-1)^{th}$ repair and the completion of the n^{th} repair on component i is called n^{th} cycle of component i for $i=1,2$ and $n=1,2,\dots$
5. Let $X_n^{(i)}$ and $Y_n^{(i)}$, $i=1,2$; $n=1,2,3,\dots$ are all independent and denotes working time and repair time respectively.
6. Let the sequence $\{X_n^{(i)}, n=1,2,3,\dots\}$ form a decreasing arithmetic geometric process, exposing to decreasing Weibull failure law with parameters $a \geq 1$ and $d_i \geq 0$
7. Let $\{Y_n^{(i)}, n=1,2,3,\dots\}$ form an increasing arithmetic geometric process exposing to an increasing Weibull failure law with parameters $0 < b < 1$ and $d_2 \leq 0$.
8. Let $E(X_n) = \mu_{X_n} > 0$ & $E(Y_n) = \mu_{Y_n} > 0$
9. Each component after repair is not as good as new.
10. A component in the system can't produce working reward during cold standby state and no cost is incurred during waiting period for repair.
11. The repair cost of two components are both C_r and the working cost of two components are both C_w and the replacement cost of the system is C .

In the next section, we discuss an optimal solution for a bivariate replacement policy (T, N) and an optimal replacement policy N based on the above assumptions.

3 OPTIMAL SOLUTIONS

In this section, we develop an optimal solution for a bivariate replacement policy (T, N) under which we replace the system when working age of component 1 reaches T and the number of failures of component 1 reaches N and an optimal replacement policy N under which we replace the system when number of failures of the component reaches N for cold standby repairable system using arithmetic – geometric process and exposing to weibull failure law. Let $C(T, N)$ be the long-run average cost per unit time under the policy (T, N) . Thus according to the renewal reward theorem of Ross [13, 14], the long-run average cost per unit time is:

$$C(T, N) = \frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of a renewal cycle}} \quad (3, 1)$$

Let L be the length of a renewal cycle of the system under policy (T, N) . Then

$$L = L_1 I_{\{U_N > T\}} + L_2 I_{\{U_N \leq T\}} \quad (3.2)$$

Where

$$L_1 = T + \sum_{n=1}^k Y_n^{(1)} + \sum_{n=2}^k (Y_n^{(2)} - X_n^{(1)}) I_{\{X_n^{(2)} - X_n^{(1)} > 0\}} + \sum_{n=1}^k (X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}} \quad (3.3)$$

Where T = total working time under policy T , second term refers repair time, third term refers waiting length of repair while the fourth term refers cold standby time of component 1 and

$$L_2 = \sum_{n=1}^N X_n^{(1)} + \sum_{n=1}^{N-1} Y_n^{(1)} + \sum_{n=2}^N (Y_n^{(2)} - X_n^{(1)}) I_{\{X_n^{(2)} - X_n^{(1)} > 0\}} + \sum_{n=1}^{N-1} (X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}} + X_N^{(2)} \quad (3.4)$$

where the first, second, third, fourth and fifth terms are respectively working time, repair time, waiting time for repair, cold standby time of component 1 under policy N and working time of component 2 under policy N , while I is the indicator function such that $I_A = 1$, if event A occurs
 $= 0$, if event A does not occur.

Now the expected length of a renewal cycle L can be evaluated as follows:

$$E(L) = E[L_1 I_{\{U_N > T\}}] + E[L_2 I_{\{U_N \leq T\}}] \quad (3.5)$$

$$E[L_1 I_{\{U_N > T\}}] = E[L_1] E[I_{\{U_N > T\}}]$$

Let

$$= \left[T + \sum_{n=1}^k Y_n^{(1)} + \sum_{n=2}^k (Y_n^{(2)} - X_n^{(1)}) I_{\{X_n^{(2)} - X_n^{(1)} > 0\}} + \sum_{n=1}^k (X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}} \right] E[I_{\{U_N > T\}}]$$

$$= E[T] E[I_{\{U_N > T\}}] + E \left[\sum_{n=2}^k (Y_n^{(2)} - X_n^{(1)}) I_{\{X_n^{(2)} - X_n^{(1)} > 0\}} \right] E[I_{\{U_N > T\}}]$$

$$+ E \left[\sum_{n=1}^k (X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}} \right] E[I_{\{U_N > T\}}] + E \left[\sum_{n=1}^N Y_n^{(1)} \right] E[I_{\{U_N > T\}}]$$

This can be evaluated by partly using the following lemma Lemma:

Let $U_N = \sum_{i=1}^N X_i^{(1)} = U_n + W_{N-n}$, $n=1,2,\dots,N-1$ then the

expectation of indicator function $I_{\{U_n < T < U_N\}}$ is given by

$$E[I_{\{U_n < T < U_N\}}] = \int_0^T \bar{F}_{N-n}(a^n(T-t)dt) F_n(t) = F_n(T) - F_N(T), \text{ where } n=1,2,\dots,N-1.$$

Where T and N are respectively the working age and the failure number of component 1 (See Leung []).

$$E[T I_{\{U_N > T\}}] = T E[I_{\{U_N > T\}}] = T \bar{F}_N(T) \quad (3.7)$$

$$E \left[\sum_{n=2}^k (Y_n^{(2)} - X_n^{(1)}) I_{\{X_n^{(2)} - X_n^{(1)} > 0\}} \right] E[I_{\{U_N > T\}}]$$

$$= \sum_{n=2}^{N-1} \int_0^\infty u g_n(u) du [F_n(T) - F_N(T)]$$

$$\sum_{n=2}^{N-1} \int_0^\infty u g_n(u) du F_n(T) - \sum_{n=2}^{N-1} \int_0^\infty u g_n(u) du F_N(T) \quad (3.8)$$

Similarly

$$E \left[\sum_{n=1}^N Y_n^{(1)} I_{\{U_N > T\}} \right] = \sum_{n=1}^{N-1} E[Y_n^{(1)}] F_n(T) - \sum_{n=1}^{N-1} E[Y_n^{(1)}] F_N(T)$$

$$\sum_{n=1}^N E[(X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}}] = \sum_{n=1}^{N-1} E[(X_n^{(2)} - Y_n^{(1)})] E[I_{\{U_N > T\}}]$$

$$= \sum_{n=1}^{N-1} \int_0^\infty v g_n(v) dv [F_n(T) - F_N(T)]$$

$$= \sum_{n=1}^{N-1} \int_0^\infty v g_n(v) dv F_n(T) - \sum_{n=1}^{N-1} \int_0^\infty v g_n(v) dv F_N(T) \quad (3.10)$$

$$\text{Let } E[L_2 I_{\{U_N \leq T\}}] = E \left[\sum_{n=1}^N X_n^{(1)} + \sum_{n=1}^{N-1} Y_n^{(1)} + \sum_{n=2}^N (Y_n^{(2)} - X_n^{(1)}) I_{\{X_n^{(2)} - X_n^{(1)} > 0\}} + X_N^{(2)} \right]$$

$$+ \sum_{n=1}^{N-1} (X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}} \cdot E[I_{\{U_N \leq T\}}] \quad (3.11)$$

This also can be evaluated by partly as follows:

$$E\left[\sum_{n=1}^N X_n^{(1)} I_{\{U_N < T\}}\right] = \sum_{n=1}^N E[X_n^{(1)}] F_N(T) \quad (3.12)$$

$$E\left[\sum_{n=1}^{N-1} Y_n^{(1)} I_{\{U_N \leq T\}}\right] = \sum_{n=1}^{N-1} E[Y_n^{(1)}] F_N(T) \quad (3.13)$$

$$E\left[\sum_{n=2}^N (Y_n^{(2)} - X_n^{(1)}) I_{\{Y_n^{(2)} - X_n^{(1)} > 0\}}\right] \cdot E[I_{\{U_N < T\}}] \\ = \sum_{n=2}^N \int_0^\infty u g_n(u) du F_N(T) \\ E\left[\sum_{n=1}^{N-1} (X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}}\right] \cdot E[I_{\{U_N \leq T\}}] \quad (3.14)$$

$$= \sum_{n=1}^{N-1} \int_0^\infty v g_n(v) dv F_N(T) \quad (3.15)$$

$$E\left[X_N^{(2)} I_{\{U_N \leq T\}}\right] = E[X_N^{(2)}] \cdot F_N(T) \quad (3.16)$$

Using equation (3.12) to (3.16), we have:

$$E[L_2 I_{\{U_N \leq T\}}] = \sum_{n=1}^N E[X_n^{(1)}] F_N(T) + \sum_{n=2}^N \int_0^\infty u g_n(u) du F_N(T) \\ + \sum_{n=1}^{N-1} \left[E(Y_n^{(1)}) + \int_0^\infty v g_n(v) dv \right] F_N(T) + E(X_N^{(2)}) F_N(T) \quad (3.17)$$

Using equation (3.7) to (3.10), we have:

$$E[L_1 I_{\{U_N > T\}}] = T \bar{F}_N(T) + \sum_{n=1}^{N-1} E(Y_n^{(1)}) [F_n(T) - F_N(T)] \\ + \sum_{n=2}^N \int_0^\infty u g_n(u) du [F_n(T) - F_N(T)] \\ + \sum_{n=1}^{N-1} \int_0^\infty v g_n(v) dv [F_n(T) - F_N(T)] \quad (3.18)$$

From equations (3.17) and (3.18), equation (3.5) becomes:

$$E[L] = T \bar{F}_N(T) + \sum_{n=1}^{N-1} E(Y_n^{(1)}) F_n(T) + \sum_{n=2}^N \int_0^\infty u g_n(u) du F_n(T) \\ + \int_0^\infty u g_N(u) du F_N(T) + \sum_{n=1}^{N-1} \int_0^\infty v g_n(v) dv F_n(T) + E(X_N^{(2)}) F_N(T) \\ + \sum_{n=1}^N E(X_n^{(1)}) F_N(T) \quad (3.19)$$

Since the failure number of component 1 reaches N, component 2 is in cold standby state in the Nth cycle or in repair state in the (N-1)th cycle.

Let S and $\bar{S}$ be such an event that component 2 is the cold standby state and repair state respectively when the working age of component 1 reaches T i.e., under policy T. And the expectations of indicator functions I_S and $I_{\bar{S}}$ are respectively given by:

$$E[I_S] = P(S) = p; P(I_{\bar{S}}) = P(\bar{S}) = q; 0 < p < 1$$

Thus, using equations (3.1) and (3.19), we have:

$$C(T, N) = \left[C_r E\left[\sum_{n=1}^K (Y_n^{(1)} + Y_n^{(2)}) I_{\{U_N > T\}}\right] I_S + \left[\sum_{n=1}^K (Y_n^{(1)} + Y_n^{(2)} - \bar{Y}_K^{(2)}) I_{\{U_N > T\}}\right] I_{\bar{S}} \right]$$

$$+ C_r E\left[\sum_{n=1}^{N-1} (Y_n^{(1)} + Y_n^{(2)}) I_{\{U_N \leq T\}}\right] + C \\ - C_w E\left[\left(T + \sum_{n=1}^K X_n^{(2)}\right) I_{\{U_N > T\}} + \left(\sum_{n=1}^N X_n^{(1)} + \sum_{n=1}^{N-1} X_n^{(2)}\right) I_{\{U_N \leq T\}}\right] \div E[L] \\ C(T, N) = \left\{ 2C_r E\left[\sum_{n=1}^K Y_n^{(1)} I_{\{U_N > T\}}\right] (p+q) - C_r q E\left[\bar{Y}_K^{(2)} I_{\{U_N > T\}}\right] \right. \\ \left. + 2C_r E\left[\sum_{n=1}^{N-1} Y_n^{(1)} I_{\{U_N > T\}}\right] + C \right. \\ \left. - C_w \left[E(T I_{\{U_N > T\}}) + E\left(\sum_{n=1}^K X_n^{(2)} I_{\{U_N > T\}}\right) \right] \right. \\ \left. + E\left(2 \sum_{n=1}^N X_n^{(1)} - X_N^{(2)}\right) I_{\{U_N < T\}} \right\} \div E[L] \quad (3.20)$$

The C (T,N) which is a bivariate function of T and N.

Where $\bar{Y}_K^{(2)}$ is excess repair time of component 2 in Kth cycle. When the working age of component 1 reaches T, the component 2 is either in the repair state in the Kth cycle or cold standby state in the (K+1)th cycle. Thus

$$\bar{Y}_K^{(2)} = Y_K^{(2)} - \left(T - \sum_{n=1}^K X_n^{(1)}\right) \\ = Y_K^{(2)} + \sum_{n=1}^K X_n^{(1)} - T \quad (3.21)$$

To evaluate the expected value of excess repair time, we determine the following probability mass function of K.

$$P[K \geq k] = P\left[\sum_{n=1}^K X_n^{(1)} < T\right] = F_K(T) \quad (3.22)$$

Where $F_K(T)$ is distribution function $\sum_{n=1}^K X_n^{(1)}$ Now

$$P[K = k] = P[K \geq k] - P[K \geq k+1]$$

From equation (2.3.19), we have:

$$P[K = k] = F_K(T) - F_{K+1}(T) \quad (3.23)$$

From equation (3.18), we have:

$$E[Y_k^{(2)} I_{\{U_N > T\}}] = E\left[\left(Y_k^{(2)} + \sum_{n=1}^K X_n^{(1)} - T\right) I_{\{U_N > T\}}\right] \quad (3.24)$$

The equation (3.21) can be evaluated partly.

By definition of conditional expectation we have:

$$E[Y_k^{(2)} I_{\{U_N > T\}}] = E\left[E(Y_k^{(2)} I_{\{U_N > T\}}) | K\right] \\ = \sum_{n=0}^N E[Y_k^{(2)} I_{\{U_N > T\}} | K = n] P(K = n) \\ = \sum_{n=0}^N E[Y_k^{(2)} I_{\{U_N > T\}}] F_n(T) - F_{n+1}(T) \\ = \bar{F}_N(T) \sum_{n=1}^N E(Y_k^{(2)}) [F_n(T) - F_{n+1}(T)] \quad (3.25)$$

$$E\left[\sum_{n=1}^K X_n^{(1)} I_{\{U_N > T\}}\right] = \sum_{n=1}^{N-1} E(X_n^{(1)}) [F_n(T) - F_N(T)] \quad (3.26)$$

According to equations (3.21) and (3.26), we have

$$E[\bar{Y}_K^{(2)} I_{\{U_N > T\}}] = \bar{F}_N(T) \sum_{n=1}^{N-1} E(Y_k^{(2)}) [F_n(T) - F_{n+1}(T)] + \\ \sum_{n=1}^{N-1} E(X_n^{(1)}) [F_n(T) - F_N(T)] - T \bar{F}_N(T) \quad (3.27)$$

Thus, using equations (3.21) to (3.27), equation (3.20) becomes:

$$C(T, N) = \left\{ 2C_r \sum_{n=1}^{N-1} E(Y_n^{(1)}) F_n(T) + C_r q \bar{F}_N(T) \sum_{n=1}^{N-1} (Y_n^{(2)} - F_n(T) - F_n(T)) \right. \\ \left. + C_r + C_r q \left[\sum_{n=1}^{N-1} E(X_n^{(1)}) F_n(T) - \sum_{n=1}^{N-1} E(X_n^{(1)}) F_n(T) + T \bar{F}_N(T) \right] \right. \\ \left. - C_w \left[T \bar{F}_N(T) + \sum_{n=1}^{N-1} E(X_n^{(2)}) F_n(T) - \sum_{n=1}^{N-1} E(X_n^{(2)}) F_n(T) \right] \right. \\ \left. + \left(2 \sum_{n=1}^N E(X_n^{(1)}) - E(X_n^{(2)}) \right) F_N(T) \right\} \div E[L] \quad (3.28)$$

Due to the assumptions of the model, $X_n^{(1)}$ and $Y_n^{(1)}$ the two random variables, denoting working time and repair time of component i in the n^{th} cycle, form a decreasing exposing to decreasing Weibull failure law and an increasing AGP exposing to an increasing Weibull failure law respectively. Therefore using the equations (3.12) to (3.15) we have:

If $X_n^{(i)} \sim W(x_n; \eta_1, \beta_1)$ the distribution functions of X_n for $i=1, 2$ is:

$$F_n(x_n) = 1 - e^{-\left[\frac{a^{n-1} x_1}{\eta_1 - (n-1) d_1 a^{n-1}} \right]^{\beta_1}}; x_1 > 0$$

Let $Y_n^{(i)} \sim W(y_n; \eta_2, \beta_2)$ then the distribution function of Y_n is:

$$F_n(y_n) = 1 - e^{-\left[\frac{b^{n-1} y_1}{\eta_2 - (n-1) d_2 b^{n-1}} \right]^{\beta_2}}; y > 0, \beta_2 > 1, i = 1, 2.$$

Then the distribution functions of $Y_{n-1}^{(2)} - X_n^{(1)}$ and $X_n^{(2)} - Y_n^{(1)}$ are

$$g(u) = L \beta_2 z^{\beta_2-1} e^{-L z^{\beta_2}}; z > 0$$

$$g(v) = K \beta_1 z^{\beta_1-1} e^{-K z^{\beta_1}}; z > 0$$

$$L = \left[\frac{b^{n-1}}{\eta_2 - (n-1) d_2 b^{n-1}} \right]^{\beta_2}$$

where

$$K = \left[\frac{a^{n-1} x_1}{\eta_1 - (n-1) d_1 a^{n-1}} \right]^{\beta_1}$$

The expected length of working time is

$$E(X_n^{(i)}) = \int_0^\infty x dF_n(x_n) = \Gamma\left(1 + \frac{1}{\beta_1}\right) \left[\frac{\eta_1}{a^{n-1}} - (n-1) d_1 \right]; i = 1, 2 \quad (3.29)$$

The expected length of repair time is:

$$E(Y_n^{(i)}) = \int_0^\infty y dG_n(y_n) = \Gamma\left(1 + \frac{1}{\beta_2}\right) \left[\frac{\eta_2}{b^{n-1}} - (n-1) d_2 \right]; i = 1, 2. \quad (3.30)$$

The expected length of waiting time for repair is:

$$E\left[(Y_{n-1}^{(2)} - X_n^{(1)}) I_{\{Y_{n-1}^{(2)} - X_n^{(1)} > 0\}}\right] = \int_0^\infty u g(u) du = \Gamma\left(1 + \frac{1}{\beta_2}\right) \left[\frac{\eta_2}{b^{n-2}} - (n-2) d_2 \right]. \quad (3.31)$$

The expected length of cold standby time is:

$$E\left[(X_n^{(2)} - Y_n^{(1)}) I_{\{X_n^{(2)} - Y_n^{(1)} > 0\}}\right] = \int_0^\infty v g(v) dv = \Gamma\left(1 + \frac{1}{\beta_1}\right) \left[\frac{\eta_1}{a^{n-1}} - (n-1) d_1 \right] \quad (3.32)$$

According to equations (3.29) to (3.32) equation (3.28) becomes:

$C(T, N) =$

$$2C_r \left\{ \Gamma\left(1 + \frac{1}{\beta_2}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_2}{b^{(n-1)}} - (n-1) d_2 \right) \right\} F_n(T) \\ + \left\{ C_r q \bar{F}_N(T) \Gamma\left(1 + \frac{1}{\beta_2}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_2}{b^{(n-1)}} - (n-1) d_2 \right) \right\} (F_{n+1}(T) - F_n(T)) + C \\ + C_r q \left\{ \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) (F_N(T) - F_n(T)) + T \bar{F}_N(T) \right\}$$

$$\left\{ T \bar{F}_N(T) + \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) (F_n(T) - F_N(T)) + \right. \\ \left. - C_w \left\{ 2 \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^N \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) - \Gamma\left(1 + \frac{1}{\beta_1}\right) \left(\frac{\eta_1}{a^{(N-1)}} - (N-1) d_1 \right) \right\} F_N(T) \right\} \\ \div E[L] \quad (3.33)$$

Where, $E[L] =$

$$T \bar{F}_N(T) + \Gamma\left(1 + \frac{1}{\beta_2}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_2}{b^{(n-1)}} - (n-1) d_2 \right) F_n(T) + \Gamma\left(1 + \frac{1}{\beta_2}\right) \sum_{n=2}^{N-1} \left(\frac{\eta_2}{b^{(n-2)}} - (n-1) d_2 \right) F_n(T) \\ + \Gamma\left(1 + \frac{1}{\beta_2}\right) \left(\frac{\eta_2}{b^{(N-2)}} - (N-1) d_2 \right) F_N(T) + \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) F_n(T) \\ + \Gamma\left(1 + \frac{1}{\beta_1}\right) \left(\frac{\eta_1}{a^{(N-1)}} - (N-1) d_1 \right) F_N(T) + \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^N \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) F_n(T) \quad (3.34)$$

This is a bivariate function of T and N.

OBSERVATIONS

i) When N is fixed $C(T, N)$ is a function of T i.e., if $N=m$, then $C(T, N) = C_m(T)$, for $m=1, 2, 3, \dots$, when $N=1, 2, 3, \dots, m, \dots$, we can find $c_1(T^*), c_2(T^*), \dots, c_m(T^*), \dots$ respectively in such way that the corresponding long-run average costs are minimum. Since the total life time of the repairable system is limited, the minimum of the long-run average cost per unit time can be determined by comparing the values of $c_1(T^*), c_2(T^*), \dots, c_m(T^*), \dots$, and it is denoted by $C(T, N) = C_n(T^*)$

ii) When

$T \rightarrow \infty$ $C(T, N) = C_2(N)$

$$\left. \begin{aligned} T \rightarrow \infty, \text{ then } L = L_2 \text{ and} \\ \lim_{T \rightarrow \infty} I_{\{Y_N > T\}} = 0, T \rightarrow \infty I_{\{Y_N \leq T\}} = 1 \\ \lim_{T \rightarrow \infty} F_n(T) = 1, T \rightarrow \infty F_N(T) = 1 \\ \lim_{T \rightarrow \infty} \bar{F}_N(T) = 0 \end{aligned} \right\} \quad (3.35)$$

Using the observation (ii) of (3.35), equation (3.33) becomes:

$$C(\infty, N) = C(N_2) = \left\{ 2C_r \Gamma\left(1 + \frac{1}{\beta_2}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_2}{b^{(n-1)}} - (n-1) d_2 \right) + C_r - C_w \left[2 \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^N \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) \right. \right. \\ \left. \left. - \Gamma\left(1 + \frac{1}{\beta_1}\right) \left(\frac{\eta_1}{a^{(N-1)}} - (N-1) d_1 \right) \right] \right\} \div E[L_2]$$

Where $E(L_2) =$

$$\Gamma\left(1 + \frac{1}{\beta_2}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_2}{b^{(n-1)}} - (n-1) d_2 \right) + \Gamma\left(1 + \frac{1}{\beta_2}\right) \sum_{n=2}^{N-1} \left(\frac{\eta_2}{b^{(n-2)}} - (n-1) d_2 \right) \\ + \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^{N-1} \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) \\ + \Gamma\left(1 + \frac{1}{\beta_1}\right) \left(\frac{\eta_1}{a^{(N-1)}} - (N-1) d_1 \right) + \Gamma\left(1 + \frac{1}{\beta_1}\right) \sum_{n=1}^N \left(\frac{\eta_1}{a^{(n-1)}} - (n-1) d_1 \right) \quad (3.36)$$

This is average cost function of N, under policy N.

In the next section, we provide numerical work to highlight the theoretical results. Using $C(N)$ we determine an optimal replacement policy N^* such that the long-run average cost per unit time is minimum.

2.4 NUMERICAL RESULTS AND CONCLUSIONS

For the hypothetical values: $a = 1.5, b = 0.5, d_1 = 1, d_2 = 1,$

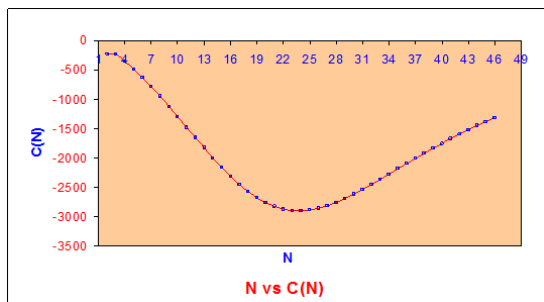
$\eta_1 = 10, \eta_2 = 20, \beta_1 = 0.5, \beta_2 = 2$

$C = 2000, C_w = 800, C_r = 10$, the optimal failure number of N^* is determined such that the long-run average cost is minimum.

Table 4.1

N	C(N)	N	C(N)	N	C(N)		
2	-225.366	16	-2310.94	30	-2614.2	44	-1448.31
3	-238.88	17	-2448.98	31	-2534.08	45	-1380.32
4	-354.931	18	-2571.09	32	-2449.46	46	-1315.3
5	-486.814	19	-2675.26	33	-2361.83	47	-1253.2

6	-631.666	20	-2760.01	34	-2272.5	48	-1193.96
7	-787.084	21	-2824.44	35	-2182.57	49	-1137.5
8	-950.984	22	-2868.33	36	-2092.93	50	-1083.71
9	-1121.42	23	-2892.04	37	-2004.33		
10	-1296.44	24	-2896.5	38	-1917.36		
11	-1473.93	25	-2883.1	39	-1832.48		
12	-1651.6	26	-2853.56	40	-1750.02		
13	-1826.9	27	-2809.85	41	-1670.24		
14	-1997.12	28	-2754.01	42	-1593.3		
15	-2159.42	29	-2688.13	43	-1519.31		



CONCLUSIONS

1. From the graph and table (4.1) we determined an optimal replacement policy $N^* = 24$ and corresponding long-run average cost is -2896.5
2. From the numerical results we observed that a small changes either 'ai' or 'bi' results a drastic changes in the optimal failure numbers of the components. Thus arithmetic-geometric process is more suitable for modeling the failure systems. Clearly as b_i increases a decreasing change in optimal failure number N.

2.6 SUMMARY AND FURTHER SCOPE

In the present paper, a little effort is done to develop an optimal replacement policy for the maintenance problems that arises in the modern industry. In this research work, we study an optimal replacement policy for a bivariate cold standby system using arithmetic-geometric process exposing to Weibull failure law by assuming that the successive working times form a decreasing arithmetic-geometric where as the consecutive repair times form an increasing arithmetic-geometric process and the system after repair is not 'as good as new' condition. Finally to highlight the theoretical results it provides analytical results.

The present work can be extended to a two dissimilar component series system and with k-dissimilar components series system and also for multistage system including k- failure states, l-working states, under the assumption that the successive working times form a decreasing arithmetic-geometric where as the consecutive repair times form an increasing arithmetic-geometric process. So this problem can be viewed as an open problem.

The present work assumed that the successive repair times form an arithmetic-geometric process. But in general the repair time can be composed with two time periods- one waiting time for repair and the other real repair time. With this assumption we can develop an optimal replacement policy for a cold standby repairable deteriorating as well as improved system.

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