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AN OPTIMAL REPLACEMENT PROBLEM FOR A DETERIORATING SYSTEM WITH INCREASING REPAIR TIMES USING ARITHMETIC GEOMETRIC PROCESSES

Statistics

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ABSTRACT

In this paper, an optimal maintenance policy for a reparable deteriorating system subject to random shocks is considered. An explicit expression for the long-term average cost per unit time under a threshold type replacement policy is determined by assuming that the successive repair time form an increasing arithmetic-geometric process and the failure mechanism by a generalized δ-shock process and minimized the long run average cost per unit time such that an optimal replacement policy N* is obtained analytically. Finally, numerical results are provided to strengthen the obtained theoretical results.

KEYWORDS

Working Reward cost, Renewal theorem, Replacement, Poisson process, Exponential process, arithmetic-geometric process, δ-Shock model.

1.INTRODUCTION

Today's modern industry has been spending corers of rupees in the maintenance scheduling with regard to allocation of available resources, improve the system reliability and maximize or minimize the total effectiveness. In this direction, from past two decades, much research work has been carried out on the maintenance problems for the system with operating and repair stages. In modeling the real systems, the assumptions with regards to repair times and working times are were not realistic in the past research. These assumptions include repaired system becoming "as good as new" and a failed system being replaced by a new one immediately. Barlow and Proschan [8] introduced an imperfect repair model, where the repair is prefect with probability 'p' and minimal with probability '1- p'. Other studies along this line include Block et al. [3], Kijima [5], Makis and Jardine [14], Dekker [7], Stadje and Zukerman [15],Braun [2], Finkelstein[6], Lam and Zhang [19] Sheu et al. [10], Lam[16-18], Zhang and Wang [20].

Many researchers made some efforts to enhance the system reliability and developed optimal replacement policies under the assumptions that the minimal repair, perfect repair, corrective repair or preventive repair and the system after repair is 'as good as new'. But for a deteriorating system it is assumed that the system after repair is not 'as good as new' and the successive repair times form decreasing **Geometric Processes (GP),** while the consecutive repair times form an increasing GP. For more details see Lam[16,17].However GP is flexible to model the repair times but in most of the cases where the failure rate cannot be approximated by straight line or inter event times drastically changes, particularly in the wear out period, the geometric process can't be more approximate. Further In such cases Leung [4] introduced an **Arithmetic-Geometric Processes(AGP)** model as more relevant, suitable and realistic model for inter event times. If the successive inter event times usually exhibit a trend, then they may be modeled by using non homogeneous processes. The non homogeneous process is one, in which the rate of time is a function of't' and varies with time. Therefore it can provide at least a good fit ordered model for a deteriorating system. If the repair time is taken into account, an arithmetic geometric process can be used an alternative to the non-homogeneous Poisson process in analyzing the data. And also for deteriorating system it can be assumed that the successive repair times form decreasing arithmetic- geometric processes (AGP), while the consecutive repair times form an increasing AGP exposing to exponential failure law. Thus, with this understanding we motivated to study an optimal maintenance policy for a reparable deteriorating system subject to random shocks using Arithmetic-Geometric Process (AGP).

Many systems are subject to random shocks from external environment in their working. Due shocking a certain amount of damage may happen to a system and eventually deteriorates the working time, or break the system down immediately. Thus, the probability of a system to survive the shocks in time interval [0, t] is a main problem in the field of shock model theory. Generally, shock models can be classified into two major types namely **1. Cumulative shock model** and **2.Extreme shock model**. In the cumulative shock model, system fails because of the cumulative effect of shocks, while in the extreme shock model, the system breaks down due to a single shock with a large amount of magnitude, see Hameed and Proschan [1], Shanthikumar and Sumita [12] for the references. On the basis of the two major types of shock models, some extensions have been developed by many researchers.

Recently, shock models were utilized to model the operating time. Conceptually, a system fails due to the shock effect on the system. While most of existing shock models were based on the accumulated or extreme damage causing a system failure, Li [21] first introduced the δ-shock model to avoid measuring amount of damage which may not be easy in many situations. Therefore, the -shock model focuses mainly on the frequency of shocks rather than the magnitude of shocks. In the -shock model, if the time interval between two consecutive shocks is smaller than a threshold value, the system fails. Thus the operating time is equal to the time to failure caused by a shock that follows the previous shock too closely. The threshold is usually set to be a constant.

Shengliang Zong, Guorong Chai, Zhe George Zhang Lei Zhao [13] considered an optimal maintenance policy for a reparable deteriorating system subject to random shocks. Modeling the repair time by a geometric process and the failure mechanism by a generalized -shock process, an explicit expression of the long-term average cost per unit time for the system under a threshold type replacement policy is determined. Based on this average cost function, they proposed an algorithm to locate the optimal replacement policy N* such that the average cost rate is minimized. Further they proved that the optimal policy N*is unique and present some numerical examples.

Based on this understanding, we study an optimal maintenance policy for a reparable deteriorating system subject to random shocks. It assumes that the successive repair time form an increasing arithmetic geometric process and the failure mechanism by a generalized -shock process. Based on these assumptions, an explicit expression of the long-term average cost per time unit for the system under a threshold type replacement policy is determined and minimized the long run average cost per unit time such that an optimal replacement policy N* is obtained analytically. Finally, numerical results are provided to strengthen the obtained theoretical results.

This paper is organized as follows. Section 1 describes the some review needed to understand the problem under considered; Section 2 presents the assumptions and definitions, Section 3 develops the long-run average cost per unit time while in Section 4, Numerical examples are provided.

2. Model definitions and assumptions

In this section, to formulate the d-shock model, we need the arithmetic-geometric process first introduced in Leung [4].

Definition 2: A Stochastic process { H_n, n=1,2,….}is an arithmetico-geometric process if there exists some real number d and some real positive number r, such that {[H_n +(n-1)d]r^{n-1} , n=1,2,…} form a renewal process (RP), where the two parameters d and b are called the common difference and common ratio of the AGP respectively.

Clearly if r >1 and d [0,/(n-1)r^{n-1}], where n=2,3,… and $_1$ is the mean of the first random variable H_1, then AGP is called a decreasing AGP. If d<0 and 0 < r < 1, then the AGP is called increasing AGP. If d=0 and r=1, then the AGP reduces to a renewal process.Thus an AGP is the name given to a series in which the general term consists of the general term of an arithmetic process (AP) and of a geometric process (GP). Thus the general term of an AGP is given by:

$$H_n = \frac{X_1}{r^{n-1}} - (n-1)d \qquad (1.1)$$

If we take d=0 but r 1 in the general term of AGP we get GP and if r=1but d 0 then we get an AP hence AGP extends and generalizes an AP or GP.

Assumptions:

1. Assume that a system is installed at t = 0. If a system fails, it is either repaired or replaced with a new and identical one.
2. Assume that the shocks arrive according to a Poisson process with rate 11λ=*iEX*
3. where X_i is the i^{th} inter-arrival time of two consecutive shocks. Let *i*δ
4. be another exponentially distributed random variable associated with Xi. We assume that the {},....3,2,1,=*ii*δ
5. forms an increasing arithmetic-geometric process. Then has cumulative distribution function)(*XHQn*
6. , where Q(x) is the cumulative distribution function of . {Xi, } follows a δ
7. -shock model if the system fails at i^{th} shock which satisfies X_i <=, and then the life time or equivalently the operating time is the sum of all X_i until the one satisfying the above condition. Further, we assume that X_i is independent of .
3. Let T_n be the operating time after the (n -1) th repair. {},.....3,2,1,=*nTn*
9. is a stochastically decreasing random variable sequence induced by theδ
10. -shock model.
4. Let Y_n be the repair time after the (n-1)th failure and forms an increasing arithmetic-geometric process .Then Y_n has cumulative distribution function)(*yHGn*
12. where G(y) is the cumulative distribution function of Y_1 with .0,1>=μμ*whereEY*13.
5. Assume that T_n and Y_n, n=1,2,3,…are two independent sequences.
6. Assume that the repair cost rate is c, the operating reward rate is r, and the replacement cost consists of fixed cost R and variable cost*Zrp*=υ
16. , where Z is the replacement time and r_p is the rate of cost per time unit during replacement.
7. Let E(Z) = t.
8. A threshold replacement policy N is adopted. Under such a policy, the system will be replaced with a new one after it fails for N times.

Due to aging process and accumulated wearing, it is reasonable to assume that the successive operating times after repairs will become shorter and shorter, and the corresponding repair times of the system will become longer and longer. Such a behavior is also be studied by arithmetic geometric process model (see Leung [4]).

This paper assumes that the successive working times forma a decreasing arithmetic-geometric process, while the consecutive shocks forma an increasing arithmetic-geometric process process and also the consecutive repair time form an increasing arithmetic-geometric process. Under the assumptions we developed an expression for the long run average cost per unit time in the following section.

3. Long-run average cost per unit time

According to renewal reward theorem (see Ross [11] and Barlow and Proschan [9]]), the long run average cost per unit time i.e., C (N) under the replacement policy N is developed as follows.

Let C(N) be the long-run average cost per unit time of the system.

$$C(N) = \frac{Expected\ cost\ incurred\ in\ a\ cycle}{Expected length\ in\ a\ cycle} \qquad (3.1)$$

Now, let W be the length of renewal cycle under the replacement N. Thus, we have

$$W = \sum_{j=1}^{N} T_j + \sum_{j=1}^{N-1} Y_j + Z \qquad (3.2)$$

To evaluate the expected cost in a cycle, we first evaluate E(Tn), the expected time of the system after the (n-1) the failure. Let l_{ni} be the inter-arrival time between (i-1)th shock and the i^{th} shock following (n-1) the repair. Where i=1,2,3,…. .

Define:

$$M_n = \min\{m / l_{n_1} > H_n\delta_1, l_{n2} > H_n\delta_1, l_{n3} > H_n\delta_1, \ldots. l_{nm} < H_n\delta_1 \qquad (3.3)$$

And

$$T_n = \sum_{i=1}^{M_n} l_{ni} \qquad (3.4)$$

Let Mn denotes the number of shocks till the first deadly shock occurs. Obviously, Mn has a Geometric distribution with probability function

$$p(M_n = k) = q_n^{k-1} p_n \ ; \ k = 1,2,3,\ldots.. \qquad (3.5)$$

Where P_n is the probability a shock following n-1th repair and q_n=1-p_n There fore we have:

$$EM_n = \frac{1}{p_n.} \qquad (3.6)$$

As M_n is stopping time with respect to the random sequence {δ},....3,2,1,=*ii*δ, which are independent and identically distributed random variables, using the Wald equation, we have

$$E(T_n) = E(\sum_{i=1}^{M_n} l_{ni}) = El_{ni} EM_n = \frac{El_{ni}}{p_n} \qquad (3.7)$$

According to the assumption (2) As F(x) and Q(x) are all identically distributed, we have

$$F(x) = 1 - \exp(\lambda_1 x), x > 0$$

$$Q(a^{n-1}x) = 1 - e - \left(\frac{a^{n-1}x}{\lambda_2 - (n-1)d_2 a^{n-1}} \right) \ , x > 0 \qquad (3.8)$$

And

$$El_{n1} = \int_0^\infty x\, dF(x) = \int_0^\infty x\, d(1 - e^{-\lambda_1 x}) = \frac{1}{\lambda_1} \qquad (3.9)$$

Further more, as $l_{ni}\ and \delta_n (H_n \delta_1)$ are independent and have the marginal distributions with means $\frac{1}{\lambda_1}\ and\ \frac{\lambda_2}{a^{n-1}} - (n-1)d_2$ respectively, we obtain

$$p_n = P(l_{ni} < \delta_n) = \int_0^\infty e^{-n^\alpha \lambda_2 x} \lambda_1 e^{-\lambda_1 x} dx$$

$$= \lambda_1 \int_0^\infty e^{-\left(\frac{a^{n-1}x}{\lambda_2 - (n-1)d_2 a^{n-1}} + \lambda_1 \right)_1 x} dx = \frac{\lambda_1}{\left(\frac{\lambda_2}{a^{n-1}} - (n-1)d_2 + \lambda_1 \right)} \qquad (3.10)$$

From equation (3.6) to equation (3.9) we have:

And $$E(T_n) = \frac{\frac{\lambda_2}{a^{n-1}} - (n-1)d_2 + \lambda_1}{\lambda_1^2} \qquad (3.11)$$

As a result of this

$$E(\sum_{n=1}^{N} T_n) = \sum_{n=1}^{N} \frac{\frac{\lambda_2}{a^{n-1}} - (n-1)d_2 + \lambda_1}{\lambda_1^2} \qquad (3.12)$$

Again , let Y_n, n=1,2,3,…. is an increasing arithmetico- geometric process then we have

$$G(b^{n-1}y) = 1 - e - \left(\frac{b^{n-1}y}{\mu - (n-1)db^{n-1}} \right) \ , y > 0$$

$$E(Y_n) = \frac{\mu}{b^{n-1}} - (n-1)d \ , \ \mu > 0. \qquad (3.13)$$

From equation (3.1), the long run average cost C(N) of the system under policy N is given by

$$C(N)=\frac{E(C\sum_{n=1}^{N-1}Y_n-r\sum_{n=1}^{N}T_n+R+r_pZ)}{E(\sum_{n=1}^{N-1}Y_n+\sum_{n=1}^{N}T_n+Z)} \tag{3.14}$$

$$C(N)=\frac{\left[C\sum_{n=1}^{N-1}EY_n-r\sum_{n=1}^{N}ET_n+R+E(r_pZ)\right]}{\sum_{n=1}^{N-1}EY_n+\sum_{n=1}^{N}ET_n+EZ)} \tag{3.15}$$

From the assumptions (6,7) and equations (3.12) and (3.13) we have:

$$C(N)=\frac{\left(C\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}-(n-1)d-r\sum_{n=1}^{N}\frac{\frac{\lambda_2}{a^{n-1}}-(n-1)d_2+\lambda_1}{\lambda_1^2}+R+r_pt\right)}{\left(\sum_{n=1}^{N-1}\frac{\mu}{b^{n-1}}-(n-1)d+\sum_{n=1}^{N}\frac{\frac{\lambda_2}{a^{n-1}}-(n-1)d_2+\lambda_1}{\lambda_1^2}+t\right)} \tag{3.16}$$

This is an expression for the long run average cost per unit time of a deteriorating system.

In the next section we determine the long run average cost per unit time with hypothetical parameters of the model.

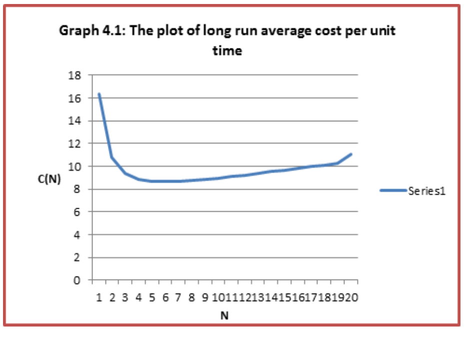
4. Numerical Results

Table: 4.1 The long run average cost per unit time is computed for the hypothetical values.

In this section, we present a numerical example to demonstrate the determination of the optimal replacement policy. Consider a system with the following parameters: a = 1.95, b=0.95, d_1=0.5, d_2=-2, c = 10, r = 400, R = 6000, t = 10, r_p = 5, $\lambda_1=5, \lambda_2=10$ *and* $\mu=20$. Us

Table 4.1: Long-run average cost per unit of time values against N

N	C(N)	N	C(N)
1	16.35617	13	9.107612
2	10.82485	14	9.247694
3	9.377013	15	9.392345
4	8.86065	16	9.539499
5	8.674581	17	9.687537
6	**8.634907**	18	9.835149
7	8.670362	19	9.981248
8	8.748655	20	10.12492
9	8.853269	21	10.26537
10	8.97485	22	11.03771



CONCLUSIONS:

From Table 4.1 and fig 4.1 it can be observed that the long run average cost per unit time is minimum i.e C(6)= **8.634907** when the number of failure N reaches 6. Similar results can be obtained with different parameter values of the model. Further if a repair man is more experiences with repair the successive repair times form a decreasing alpha series process. Therefore, we can develop an optimal replacement policy N for an improved system.

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