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AN OPTIMAL REPLACEMENT PROBLEM FOR A SHOCK MODEL WITH DIFFERENT MODES OF FAILURES USING ALPHA SERIES PROCESSES

Statistics

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ABSTRACT

This paper studies a shock model for a repairable system with two-type failures by assuming that two kinds of shocks in a sequence of random shocks will make the system failed, one based on the inter-arrival time between two consecutive shocks less than a given positive value and the other based on the shock magnitude of single shock more than a given positive value . Further it is assumed that the system after repair is not 'as good as new', but the consecutive repair times of the system form a stochastic increasing α-seires process. Under these assumptions, we determine an explicit expression for the average cost rate and an optimal placement policy N* based on the number of failure of the system is determined such that the long-run average cost per unit time is minimized. The explicit expression of long-run average cost per unit time is derived, and the corresponding optimal replacement policy can be determined analytically or numerically. Finally, a numerical example is given.

KEYWORDS

Replacement policy, α-series process, renewal reward theorem, Average cost rate, MTTF.

1. INTRODUCTION

Most repair replacement models assume that a failure system after repair will yield a function system which is **"as good as new"** and the repair times are neglected, so that the successive operating times forms a **renewal process.** These types of models may be called as '**perfect repair models'**.

Barlow and Hunter [1] studied a minimal repair model in which a minimal repair does not change the age of the system. Thereafter an imperfect repair model was developed by Brown and Proschan [3] under which a repair with probability **'p'** as perfect repair and with probability **'1-p'** as minimal repair. Many others worked in this direction and developed corresponding optimal replacement polices by Black et.al [2], Finkelstein [5], Stadje and Zukerman [12], Stanley [13] , Lam[6,9], Lam and Zhang [10] and Zhang and Wang [17].

In general, for a deteriorating system, it is reasonable to assume that the successive working times are stochastically decreasing while the consecutive repair times after failures are stochastically increasing, due to the ageing and accumulated wearing in many systems.To model such simple repairable deteriorating system.

A repair may occur due to either internal shock or external shock for a repairable system. The shock model is one of the important problems in reliability and maintenance theory. It is mainly used to study the external causes such as the environmental factor which may influence a system. For example, in a power generator a shock may be affected by some random causes or due to the operation of other instruments, or electrical machineries.

Wang and Zhang [14] considered a shock model for a repairable system with two types of failures by assuming that two kinds of shock in a sequence of random shocks will make the system failed, one based on the inter arrival time between two consecutive shocks less than given positive and the other based on the shock magnitude of single shock more than a given positive value (Ψ). Under this assumption they obtained some reliability indices of the shock model such as the system reliability and the mean working time before system failure. Also determined the replacement policy N based on the number of failures of the system by minimizing the long-run average cost per unit time. Further, it is also established through numerically.

For a deteriorating system, a more reasonable repair model is the geometric process repair model first proposed by Lam [7,8]. Under this model, Lam studied two kinds of replacement policy, one based on the working age T of the system and the other based on the failure number N of the system. The object is to choose optimal replacement policies T* and N* respectively such that the long-run average cost per unit time is minimized. The explicit expressions for the long-run average cost per unit time under these two kinds of policy are respectively derived, and the corresponding optimal replacement policies T* and N* are found analytically or numerically. Because the geometric process is a special monotone process, Stadje and Zuckerman [12] introduced a general monotone process repair model to generalize Lam's work.

Zhang [15] generalized Lam's work by a bivariate replacement policy (T, N) under which the system is replaced at the working age T or at the time of the N^{th} failure, whichever occurs first. At the same time, for improving the system reliability or raising the system availability, Zhang [16] applied the geometric process repair model to a two-component cold standby system with one repairman. Assuming that each component after repair is not 'as good as new', by using the geometric process, he considered a replacement policy N based on the number of repair of component 1. The problem is to determine the optimal replacement policy N* such that the long-run expected reward per unit time s maximized.

Brown et.al [4] studied some important properties of monotone processes and proved that alpha series processes is more appropriate to model the up times. On this understanding, in this chapter we have proposed to develop alpha series processes maintenance model and obtained an optimal replacement policy.

In this paper, a shock model for a repairable system with two-type failures is studied. It assumes that two kinds of shock in a sequence of random shocks will make the system failed, one based on the interarrival time between two consecutive shocks less than a given positive value δ (i.e. so-called 'δ-shock model') and the other based on the shock magnitude of single shock more than a given positive value δ (i.e. so-called 'δ- shock model'). Assume further that the system after repair is 'as good as new' while the consecutive up times of the system will become longer and longer. Under these assumptions, by using the α-series process repair model , the replacement policy N based on the number of failure of the system is determined. The problem is to obtain an optimal replacement policy N* such that the long-run average cost per unit time is minimized. The explicit expression of the long-run average cost per unit time is derived, and hence an optimal policy N* can be determined analytically or numerically. Finally, a numerical example is given.

2. THE δ-SHOCK MODEL

When a repairable system is working, it may be failed subject to an external cause such as random shock. In this paper, we consider two kinds of random shock such that the system is failed. Let T_n and D_n denote respectively the time interval between (n-1)-th shock and nth shock, and the magnitude of nth shock.

And let δ and γ be two positive in the system. The system will fail as soon as $Tn<\delta$ appears, n=1, 2 . . . and the system failure caused by $T_n<\delta$ is called the first type failure. In fact, it is a δ-shock model considered by Wang and Zhang (2001). Similarly, the system will fail as soon as $D_n>\gamma$, n=1, 2 . . . appears and the system failure caused by $D_n>\gamma$ is called the second type failure. In fact, it is a shock model studied by Shanthikumar and Sumita (1983). Note that the system will fail as soon as both $T_n<\delta$ and $D_n>\gamma$ appear n=1, 2, ... Thus, we may as well call the system failure the third type failure. In other words, two-type failures will cause three cases of the system failure. It is a more extensive shock model in which we not only consider the magnitude of single shock but also the time interval of consecutive two shocks to theeffect of the system. Obviously, the model in this paper will be degenerated into the model in Wang and Zhang(2001) when $\gamma\to\infty$while it will be returned to the model in Shanthikumar and Sumita (1983) when $\delta\to 0$.

Let $S_1, S_2, \ldots$ be shock arrival time when the system is working, and assume that

$\{T_n, n=1, 2, \ldots\}$ and $\{D_n, n=1, 2, \ldots\}$ are respectively a sequence of i.i.d. non-negative random variables.

Clearly,,,......2,1,......21=++=*nTTTSnn* (2.1)

Let X be the working time before the system first fails, then

$$X=T_1+T_2+\ldots\ldots+T_{M,} \tag{2.2}$$

where M is shock number of times at the system failure, and it is a random variable. Now, we give out some results of the system under shock model with two-type failure. According to Definition 3, $\{T_n, n=1, 2, \ldots\}$ and $\{D_n, n=1, 2, \ldots\}$ are respectively a sequence of i.i.d. non-negative random variables. We might as well let T and D denote respectively the time interval of consecutive two shocks and the magnitude of single shock.

Theorem 1: Assume that $\{(T_n, D_n), n = 1, 2, \ldots\}$ forms a correlated renewal sequence pair with joint distribution function

$$F(x, y) = P(T_n \le x,\ D_n \le y)$$

and H(t) is the distribution function of X, then the Laplace transform of H(t) is given by

$$H^*(s) = \frac{F_T^*(s) - F^*(s, \gamma)}{1+\int_0^{\delta} e^{-sx}\, dF(x,\gamma) - F^*(s, \gamma)} \tag{2.3}$$

where $F_T^*(s) = \int_0^{\infty} e^{-sx} F_T(x)\, dx$, $F^*(s, \gamma) = \int_0^{\infty} e^{-sx}\, dF_T(x, y)$, δ and γ are two given positive parameters

Thus, we can get the system reliability R(t) = $\overline{H}$ (t) or its Laplace transform R*(s) from Theorem 1.

Let

$$P_{00} = P(T > \delta, D < \gamma) \tag{2.4}$$

$$P_{10} = P(T < \delta, D < \gamma) \tag{2.5}$$

$$P_{01} = P(T > \delta, D > \gamma) \tag{2.6}$$

$$P_{11} = P(T < \delta, D > \gamma) \tag{2.7}$$

denote respectively the probabilities that neither the first type failure nor the second type failure happens, only the first type failure happens, only the second type failure happens and both the two-type failures happen in the system. Clearly, $p_{00}+p_{10}+p_{01}+p_{11}=1$.

Theorem 2: Under the condition of Theorem 1 given by G. J. Wang & Y. L. Zhang [14], the mean working time before the system fails is given by :

$$EX = \frac{ET}{1-p_{00}}. \tag{2.8}$$

The following **assumptions** about the repairable system for random shock with two-type failures are considered.

1. At the beginning, the system is new. The system will be repaired as soon as it failed. The system will be replaced some time by a new and identical one, and the replacement time is negligible.
2. Assume that the system after repair is 'as good as new' while the consecutive repair times of the system form a stochastic increasing alpha process. The time interval between the completion of the (n-1)-th repair and the completion of the n-th repair on the system is called the n-th cycle of the system, n=1, 2, . .
3. Let X_n and $Y^{(i)}_n$ be respectively the working time and the repair time of the system incurred by i-th type failure in the n-th cycle, i=1, 2, 3; n=1, 2, ... let $\{X_n, n=1, 2, \ldots\}$ is a sequence of the i.i.d. non-negative random variables with distribution function, $F(n^{\alpha} t)$ while the distribution of $Y^{(i)}_n$ is assumed to be $G(n\beta t)$, and the assume that ET1=λ and E =μi , where $t\ge 0$, 0<bi<1, $\lambda>0$, $\mu i>0$; i=1, 2, 3; n=1, 2,
4. Assume that Xn, , i=1, 2, 3; n=1, 2, . . . are independent.
5. The replacement policy N under which the system is replaced when the failure number of the system is reaches N is considered.
6. Assume that the repair cost incurred by i-th type failure of the system per unit time is $C_r^{(i)}$, i=1, 2, 3, the working reward of the system per unit time is cw, and the replacement cost of the system is C.

3. LONG-RUN AVERAGE COST UNDER POLICY N

To determine an optimal replacement policy N such that the long-run average cost per unit time is minimum let $\tau 1$ be the first replacement time, and let $\tau n(n>1)$ be the time between the (n-1)-th replacement and the n-th replacement. Obviously, $\{\tau 1, \tau 2, \tau 3, \ldots\}$ forms a renewal process. Let C(N) be the long-run average cost per unit time under the replacement policy N. Because $\{\tau 1, \tau 2, \tau 3, \ldots\}$ is a renewal process, the time interval between two consecutive replacements is a renewal cycle. Then according to the renewal reward theorem (see, for example, Ross [11], we have

$$C(N) = \frac{\text{the expected cost incurred in a cycle}}{\text{the expected length of a cycle}}. \tag{3.1}$$

where the length of the renewal cycle is given by

$$W = X_1 + \left[Y_1^{(1)} I_1^{(1)} + Y_1^{(2)} I_1^{(2)} + Y_1^{(3)} I_1^{(3)}\right] + X_2 + \left[Y_2^{(1)} I_2^{(1)} + Y_2^{(2)} I_2^{(2)} + Y_2^{(3)} I_2^{(3)}\right] + \ldots\ldots\ldots + X_{N-1} + \left[Y_{N-1}^{(1)} I_{N-1}^{(1)} + Y_{N-1}^{(2)} I_{N-1}^{(2)} + Y_{N-1}^{(3)} I_{N-1}^{(3)}\right] + X_N. \tag{3.2}$$

To evaluate EW, we only need to calculate the following results by using the property of the conditional expectation

$$E(Y_n^{(1)} I_n^{(1)}) = E\left[E(Y_n^{(1)} I_n^{(1)} \mid I_n^{(1)})\right] = P(I_n^{(1)} = 1)\, E(Y_n^{(1)}) \tag{3.3}$$

$$E(Y_n^{(2)} I_n^{(2)}) = E\left[E(Y_n^{(2)} I_n^{(2)} \mid I_n^{(2)})\right] = P(I_n^{(2)} = 1)\, E(Y_n^{(2)}) \tag{3.4}$$

$$E(Y_n^{(3)} I_n^{(3)}) = E\left[E(Y_n^{(3)} I_n^{(3)} \mid I_n^{(3)})\right] = P(I_n^{(3)} = 1)\, E(Y_n^{(3)}) \tag{3.5}$$

Let q_i, i=1, 2, 3 be the probability of the i-th type failure when failure happen, then

$$E(I_n^{(1)}) = q_1 = P(I_n^{(1)}=1) = \frac{P_{10}}{1-P_{00}} \tag{3.6}$$

$$E(_n^{(2)}) = q_2 = P(I_n^{(2)}=1) = \frac{P_{01}}{1-P_{00}} \tag{3.7}$$

$$E(I_n^{(3)}) = q_3 = P(I_n^{(3)}=1) = \frac{P_{11}}{1-P_{00}}. \tag{3.8}$$

Clearly, $q_1+q_2+q_3=1$; and

According to assumption of the model, we have

$$E(Y_n^{(i)}) = \int_0^{\infty} y\, dG_n(n^{\beta} y) = \frac{\mu_i}{n^{\beta}}, i=1,2,3.$$

$$E(X_n^{(i)}) = \int_0^{\infty} x\, dF_n(n^{\beta} x) = \frac{\lambda}{n^{\beta}}.$$

$$E(Y_n^{(1)} I_n^{(1)}) = \frac{\mu_1}{n^{\beta_1}}\cdot q_1 = \frac{\mu_1}{n^{\beta_1}} \frac{P_{10}}{1-P_{00}} \tag{3.9}$$

$$E\left[Y_n^{(2)} I_n^{(2)}\right] = \frac{\mu_2}{n^{\beta_2}} . q_2 = \frac{\mu_2}{n^{\beta_2}} \frac{P_{01}}{1-P_{00}} \tag{3.10}$$

$$E\left[Y_n^{(3)} I_n^{(3)}\right] = \frac{\mu_3}{n^{\beta_3}} . q_3 = \frac{\mu_3}{n^{\beta_3}} \frac{P_{11}}{1-P_{00}} \tag{3.11}$$

According to the assumption (4) of the model and using equations (3.9) to (3.11), the expected length of a renewal cycle is

$$EW = NEX + \mu_1 q_1 \sum_{n=1}^{N-1} \frac{1}{n^{\beta_1}} + \mu_2 q_2 \sum_{n=1}^{N-1} \frac{1}{n^{\beta_2}} + \mu_3 q_3 \sum_{n=1}^{N-1} \frac{1}{n^{\beta_3}} \tag{3.12}$$

$$= \frac{\lambda}{1-p_{00}} \sum_{n=1}^{N} \frac{1}{n^{\alpha}} + \mu_1 \frac{p_{10}}{1-p_{00}} \sum_{n=1}^{N-1} \frac{1}{n^{\beta_1}} + \mu_2 \frac{p_{01}}{1-p_{00}} \sum_{n=1}^{N-1} \frac{1}{n^{\beta_2}} + \mu_3 \frac{p_{11}}{1-p_{00}} \sum_{n=1}^{N-1} \frac{1}{n^{\beta_3}} \tag{3.13}$$

According to equations (3.1) and (3.2), we have

$$C(N) = \frac{c_r^{(1)} \mu_1 q_1 \sum_{n=1}^{N-1} E(Y_n^{(1)}) + c_r^{(2)} \mu_2 q_2 \sum_{n=1}^{N-1} E(Y_n^{(2)}) + c_r^{(3)} \mu_3 q_3 \sum_{n=1}^{N-1} E(Y_n^{(3)}) + C - c_w\, NEX}{EW}$$

According to equations (3.3) and (3.11), we have

$$= \frac{c_r^{(1)} \mu_1 \left(\frac{p_{10}}{1-p_{00}}\right) l_1 + c_r^{(2)} \mu_2 \left(\frac{p_{01}}{1-p_{00}}\right) l_2 + c_r^{(3)} \mu_3 \left(\frac{p_{11}}{1-p_{00}}\right) l_3 + C - C_w \lambda \left(\frac{1}{1-p_{00}}\right) \sum_{n=1}^{N} \frac{1}{n^{\alpha}}}{\lambda \left(\frac{1}{1-p_{00}}\right) \sum_{n=1}^{N} \frac{1}{n^{\alpha}} + \mu_1 \frac{p_{10}}{1-p_{00}} \sum_{n=1}^{N-1} \frac{1}{n^{\beta_1}} + \mu_2 \frac{p_{01}}{1-p_{00}} \sum_{n=1}^{N-1} \frac{1}{n^{\beta_2}} + \mu_3 \frac{p_{11}}{1-p_{00}} \sum_{n=1}^{N-1} \frac{1}{n^{\beta_3}}}$$

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Where $l_1 = \sum_{n=1}^{N-1} \frac{1}{n^{\beta_1}}$ $l_2 = \sum_{n=1}^{N-1} \frac{1}{n^{\beta_2}}$ $l_3 = \sum_{n=1}^{N-1} \frac{1}{n^{\beta_3}}$

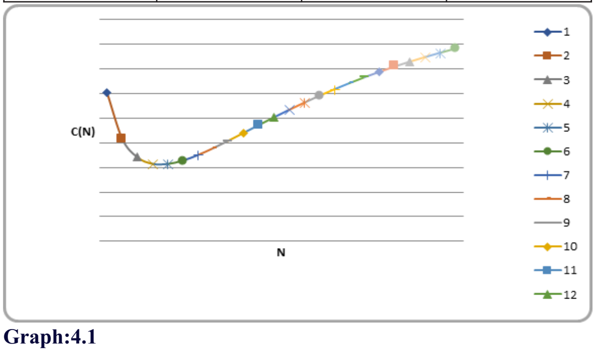
In the next section, numerical results are provided to highlight the obtained theoretical results.

4.4 NUMERICAL RESULTS AND CONCLUSIONS:

Table4.1

For the given hypothetical values of the parameters $\mu_1, \mu_2, \mu_3, \lambda, C_r^{(1)}, C_r^{(2)}, C_r^{(3)}, q_1, q_2, q_3, \beta_1, \beta_2, \beta_3$, C ,and C_w we determine the long run average cost per unit time from the equation (9) as followsWhere μ_1=50, μ_2=60, μ_3=70, λ=30, $C_r^{(1)}$ =10, $C_r^{(2)}$=20, $C_r^{(3)}$=30, q_1=0.25, q_2=0.35, q_3=0.40, β_1=-0.65, β_2=-0.75, β_3=-0.95, α=0.95,C=45000 ,and C_w=30.

N	C(N)	N	C(N)
1	30.19305	13	26.70477
2	20.83223	14	28.17112
3	17.07673	15	29.57233
4	15.75637	16	30.90444
5	15.71734	17	32.16618
6	16.41338	18	33.35808
7	17.54337	19	34.48188
8	18.92553	20	35.54014
9	20.44416	21	36.53584
10	22.02355	22	37.47227
11	23.61356	23	38.35279
12	25.18111	24	39.18077



Graph:4.1

Conclusions:

From the table 4.1 and graph 4.1, it is identified that the long-run average cost per unit time is minimized when the number of failure of the system reaches 6.i.e., C(5)=- 15.71734 at μ_1=50, μ_2=60, μ_3=70, λ=30, $C_r^{(1)}$ =10, $C_r^{(2)}$=20, $C_r^{(3)}$=30, q_1=0.25, q_2=0.35, q_3=0.40 , β_1=-0.65, β_2=-0.75, β_3=-0.95, α=0.95,C=45000 ,and C_w=30.

From the above conclusions, it is examined that values of the parameters of the process are inversely related with the long run average cost. This is also coinciding with practical analogy.

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